

Renormalization of MSSM

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Beijing,
May 5, 2015

1 What is renormalization

- Remove the divergence in the loop amplitudes, respecting gauge invariance(BRS invariance)
- Give the physical meaning to the parameters appearing in the lagrangian.

Example:QED

$$\mathcal{L}^{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\Psi}(i\not{D} - e\not{A} - m)\Psi$$

On-shell renormalization \rightarrow m is the electron mass.

$\overline{\text{MS}}$ renormalization $\rightarrow m + \overline{\delta m}$ is the electron mass.

2 General feature of the renormalization

- In general, one can put the renormalization constants to all the fields and parameters appearing in the lagrangian.
- Not all of them are independent.
- Some of them are redundant.
The wavefunction renormalization constants of the unphysical particles automatically cancel out in the sum of the amplitude.
- The wavefunction renormalization constants of the physical particles can be 1 ($\delta Z = 0$) when we treat the sum of the amplitude of physical processes (A. Sirlin).
- The redundant renormalization constants can be used to simplify the amplitude computations.
- Process dependent renormalization conditions vs. process independent renormalization conditions
- Scheme dependence comes out of the choice of the renormalization conditions.

Example

1. Residue condition: the residue of the propagator at the pole is 1.
→ δZ is fixed.
→ This condition leads to the vanishing of the selfenergy insertion in the external line.
2. Condition of vanishing of the transition (mixing) $A \rightarrow B$ for on-shell A .
→ The external line correction of A , $A \rightarrow B$, vanishes.

3 Technical complications of the renormalization of MSSM

1. The light CP even Higgs is lighter than Z at tree level.
 → Large radiative corrections
 → One cannot use the Higgs mass (126 GeV object) as input mass.
2. More particles than the number of mass parameters in the lagrangian.
 → Not all the particle masses are independent.
 → On-shell mass condition is not imposed on some particles.
3. None of the mass of the SUSY particles is known.
 The situation is similar to $\cos \theta_W$ of the SM in the early 1970's.
 → On which particles should we impose the on-shell condition?
4. Mixing of four neutral Weyl spinors
 → Four-by-four mass matrix of neutralinos.
 → Difficult to find a counterterm of the mass matrix
5. The number of the interaction types is more than the number of parameters in the lagrangian.
 Similar to the QCD coupling g , which appears in $q\bar{q}g$ (Yukawa coupling), ggg (triple gauge coupling), or $gggg$ (quartic gauge coupling).

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \bar{\Psi}(\not{\partial} - gT^a A - m)\Psi - \frac{1}{2\xi}(\partial_\mu A^{a\mu})^2 + \text{ghost terms}.$$

6. Majorana particles(Fermion number is not conserved)

4 MSSM Lagrangian (Electroweak sector)

4.1 particles in the lagrantian

gauge bosons	$\vec{W}_\mu, B_\mu,$
Higgs doublets	$H_1 = \begin{pmatrix} \frac{v_1 + \phi_1^0 - i\chi_1^0}{\sqrt{2}} \\ -\phi_1^- \end{pmatrix}, H_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + \phi_2^0 + i\chi_2^0}{\sqrt{2}} \end{pmatrix},$
conventional fermions	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R, \dots\dots$
gauginos(Weyl spinors)	$\vec{\lambda}, \lambda$
Higgsinos(Weyl spinors)	$\tilde{H}_1^0, \tilde{H}_2^0, \tilde{H}_1^-, \tilde{H}_2^+,$
sfermion	$\tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R, \dots\dots,$

4.2 physical particles

gauge bosons	$W_\mu^\pm, Z_\mu, A_\mu,$
Higgs	$h^0, H^0, A^0, H^\pm,$
conventional fermions	$u, d, c, s, t, b, e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau,$
chargino(Dirac)	$\chi_1^\pm, \chi_2^\pm,$
neutralinos(Majorana)	$\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0$
sfermions	$\tilde{u}_1, \tilde{u}_2, \dots\dots,$

4.3 unphysical particles

would – be Goldstone bosons	$G^\pm, G^0,$
ghosts	$\omega^\pm, \omega^z, \omega^\gamma, \bar{\omega}^\pm, \bar{\omega}^z, \bar{\omega}^\gamma$

4.4 mixing

$$\begin{aligned}
\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} &= \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \\
\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} &= \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \\
\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} &= \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \\
\begin{pmatrix} G^\pm \\ H^0 \end{pmatrix} &= \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}.
\end{aligned}$$

4.5 gauge fixing

R_ξ gauge is adopted in order to remove the gauge-boson Goldstone-boson transition terms in the Higgs potential.

Gauge symmetric states:

$$\begin{aligned}
\mathcal{L}_{g.f.} &= -\frac{1}{2\xi_W}|F^a|^2 - \frac{1}{2\xi_B}|F^B|^2, \\
F_\mu^a &= \partial_\mu W^{a\mu} - ig\xi_W \left(\mathbf{H}_i^\dagger \frac{\tau^a}{2} \langle \mathbf{H}_i \rangle - \langle \mathbf{H}_i \rangle^\dagger \frac{\tau^a}{2} \mathbf{H}_i \right), \\
F^B &= \partial_\mu B^\mu + i\frac{g'}{2}\xi_B \left(\mathbf{H}_i^\dagger \langle \mathbf{H}_i \rangle - \langle \mathbf{H}_i \rangle^\dagger \mathbf{H}_i \right),
\end{aligned}$$

Explicitly

$$\begin{aligned}
\mathcal{L}_{g.f.} &= -\frac{1}{\xi_W}|\partial_\mu W^{\pm\mu}|^2 - \xi_W M_W^2 G^+ G^- \\
&\quad -\frac{1}{2\xi_W}(\partial_\mu W^{3\mu})^2 - \frac{\xi_W}{2} M_W^2 G^0 G^0 \\
&\quad -\frac{1}{2\xi_B}(\partial_\mu B^\mu)^2 - \frac{\xi_B}{2} s_W^2 M_Z^2 G^0 G^0 \\
&\quad + (\text{Goldstone boson gauge boson transition terms})
\end{aligned}$$

Mass eigenstates:

$$\begin{aligned}
\mathcal{L}_{g.f.} &= -\frac{1}{\xi_W}|F^\pm|^2 - \frac{1}{2\xi_Z}|F^Z|^2 - \frac{1}{\xi_A}|F^A|^2, \\
F_\mu^\pm &= \partial_\mu W^{\pm\mu} \pm ig\xi_W M_W G^\pm, \\
F^Z &= \partial_\mu Z^\mu + \xi_Z M_Z G^0, \\
F^A &= \partial_\mu A^\mu.
\end{aligned}$$

4.6 parameters in the lagrangian

SUSY	$g, g', v_1, v_2, \mu, m_f$	
SUSYbreaking	$m_1^2, m_2^2, m_{12}^2,$	Higgs mass
	$M_1, M_2, A_f,$	chargino, neutralino mass
	$\tilde{m}_{f_L}^2, \tilde{m}_{f_R}^2,$	sfermion mass

4.7 Parameters used in the renormalization

$$\begin{aligned}
g, g', v_1, v_2, m_1^2, m_2^2, m_{12}^2 &\rightarrow e, M_W, M_Z, M_{H^\pm}, \tan \beta, T_{\phi_1^0}, T_{\phi_2^0}, \\
A_e, \tilde{m}_{e_L}^2 = \tilde{m}_{\nu_L}^2, \tilde{m}_{e_R}^2 &\rightarrow m_{\tilde{e}_L}^2, m_{\tilde{e}_R}^2, m_{\tilde{e}_{LR}}^2, (m_{\tilde{\nu}}^2) \\
A_t, A_b, \tilde{m}_{t_L}^2 = \tilde{m}_{b_L}^2, \tilde{m}_{t_R}^2, \tilde{m}_{b_R}^2 &\rightarrow m_{\tilde{t}_L}^2, m_{\tilde{t}_R}^2, m_{\tilde{t}_{LR}}^2, (m_{\tilde{b}_L}^2) m_{\tilde{b}_R}^2, m_{\tilde{b}_{LR}}^2
\end{aligned}$$

where

$$\begin{aligned}
e &= \frac{gg'}{\sqrt{g^2 + g'^2}}, \\
M_W^2 &= \frac{g^2}{4}(v_1^2 + v_2^2), \\
M_Z^2 &= \frac{g^2 + g'^2}{4}(v_1^2 + v_2^2), \\
M_{H^\pm}^2 &= m_1^2 + m_2^2 + 2\mu^2 + M_W^2, \\
\tan \beta &= \frac{v_2}{v_1}, \\
T_{\phi_1^0} &= (m_1^2 v_1 + m_{12}^2 v_2 + \frac{g^2 + g'^2}{8}(v_1^2 - v_2^2)v_1), \\
T_{\phi_2^0} &= (m_2^2 v_2 + m_{12}^2 v_1 - \frac{g^2 + g'^2}{8}(v_1^2 - v_2^2)v_2), \\
m_{\tilde{f}_L}^2 &= \tilde{m}_{f_L}^2 + m_f^2 + M_Z^2 \cos 2\beta (T_{3f} - Q_f s_W^2), \\
m_{\tilde{f}_R}^2 &= \tilde{m}_{f_R}^2 + m_f^2 + M_Z^2 \cos 2\beta Q_f s_W^2, \\
m_{\tilde{f}_{LR}}^2 &= \begin{cases} m_f(\mu \cot \beta - A_f), & f = t \\ m_f(\mu \tan \beta - A_f), & f = b, \tau \end{cases}
\end{aligned}$$

- $M_A^2 = m_1^2 + m_2^2 + 2\mu^2$ can be used as an input in place of $M_{H^\pm}^2$.
HOWEVER
- Apply the on-shell condition on as many charged particles as possible.
External soft photon radiation \leftrightarrow Infrared singularity in the loop amplitude
- We cannot use tree equation for M_h^0

$$M_{h^0}^2 = \frac{1}{2} \left(M_A^2 + M_Z^2 - \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta} \right)$$

to fix one of the parameters ($\tan \beta$) appearing in the lagrangian, although we would like to identify the object of 126 GeV as h^0 .

- The v.e.v (v_1, v_2) are determined by the potential minimum condition,

$$T_{\phi_1^0} = T_{\phi_2^0} = 0.$$

5 Renormalization

5.1 Renormalization constants

- Masses and couplings

$$\begin{aligned}
g_0 &= Z_g Z_W^{-3/2} g, \\
g'_0 &= Z_{g'} Z_B^{-3/2} g', \\
v_{i0} &= Z_{H_i}^{1/2} (v_i - \delta v_i), \\
m_{i0}^2 &= Z_{H_i}^{-1} (m_i^2 + \delta m_i^2), i = 1, 2 \\
m_{120}^2 &= Z_{H_1}^{-1} Z_{H_2}^{-1} (m_{12}^2 + \delta m_{12}^2),
\end{aligned}$$

$$\begin{aligned}
(m_{\tilde{f}_L}^2)_0 &= m_{\tilde{f}_L}^2 + \delta m_{\tilde{f}_L}^2, \\
(m_{\tilde{f}_R}^2)_0 &= m_{\tilde{f}_R}^2 + \delta m_{\tilde{f}_R}^2, \\
(m_{\tilde{f}_{LR}}^2)_0 &= m_{\tilde{f}_{LR}}^2 + \delta m_{\tilde{f}_{LR}}^2,
\end{aligned}$$

$$\begin{aligned}
\mu_0 &= \mu + \delta\mu, \\
M_{10} &= M_1 + \delta M_1, \\
M_{20} &= M_2 + \delta M_2,
\end{aligned}$$

- the wavefunction renormalizations of gauge fields
two options: before SSB or after SSB.
before SSB: (There exists no counterterm of $\cos \theta_W$.)

$$\begin{aligned}
\vec{W}_{\mu 0} &= Z_W^{1/2} \vec{W}_\mu, \\
B_{\mu 0} &= Z_B^{1/2} B_\mu,
\end{aligned}$$

after SSB:

$$\begin{aligned}
W_{\mu 0}^\pm &= Z_W^{1/2} W_\mu^\pm, \\
\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}_0 &= \begin{pmatrix} Z_{AA}^{1/2} & Z_{AZ}^{1/2} \\ Z_{AZ}^{1/2} & Z_{ZZ}^{1/2} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}.
\end{aligned}$$

- The wavefunction renormalization constants of Higgs

$$\mathbf{H}_{i0} = Z_{H_i}^{1/2} \mathbf{H}_i.$$

- Ino sector

Four neutral Weyl spinors \rightarrow four neutralinos (Majorana particles).

It is preferable not to diagonalize the neutralino mass matrix in the bare lagrangian. If we diagonalize the mass matrix in the bare lagrangian with mixing angles θ 's, we have to compute the counterterm $\delta\theta$ in terms of the parameters appearing in the lagrangian, which is rather difficult.

We diagonalize the neutralino mass matrix in the renormalized lagrangian. \rightarrow no counterterm for the mixing angles.

$$\begin{aligned}\vec{\lambda}_0 &= Z_{\lambda_w}^{1/2} \vec{\lambda}, \\ \lambda_0 &= Z_{\lambda}^{1/2} \lambda, \\ \tilde{\mathbf{H}}_{i0} &= Z_{\tilde{H}_i}^{1/2} \tilde{\mathbf{H}}_i, \quad i = 1, 2.\end{aligned}$$

- sfermion

$$\begin{aligned}\begin{pmatrix} \tilde{\nu} \\ \tilde{\ell}_L \end{pmatrix}_0 &= Z_L^\ell \begin{pmatrix} \tilde{\nu} \\ \tilde{\ell}_L \end{pmatrix}, \quad (\tilde{\ell}_R)_0 = Z_R^\ell \tilde{\ell}_R, \\ \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix}_0 &= Z_L^t \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix}, \quad (\tilde{t}_R)_0 = Z_R^t \tilde{t}_R, \quad (\tilde{b}_R)_0 = Z_R^b \tilde{b}_R.\end{aligned}$$

- gauge parameters

$$\xi_{V0} = Z_{\xi_V}^{1/2} \xi_V, \quad V = W, B \text{ or } W^\pm, Z, A$$

5.2 On-shell Renormalization

- Fermion
Identical to SM
- Tadpole ($\delta T_{h^0}, \delta T_{H^0}$)
Identical to SM

$$\delta T_{h^0} = T_{h^0}^{loop}, \quad \delta T_{H^0} = T_{H^0}^{loop}$$

- Gauge sector ($\delta M_W^2, \delta M_Z^2, \delta Z_W, \delta Z_B$, one constraint containing δZ_{H_i} etc)
Similar to SM
 $U_{EM}(1)$ gauge invariance $\rightarrow \delta Z_B = \delta Z_{g'}$
On-shell mass condition for W^\pm and Z .
Residue condition for photon
Decoupling of A_μ and Z_μ for on-shell photon ($q^2 = 0$)
Recall that the residue conditions for W and Z propagator

$$\begin{aligned} \delta Z_W &= \Pi'_{WW}(M_W^2) \\ c^2 \delta Z_W + s^2 \delta Z_B &= \Pi'_{ZZ}(M_Z^2) \end{aligned}$$

violates the gauge invariance (D.Ross and J.C.Taylor; J.P.Cole).

- Gauge fixing and ghost ($\delta Z_{\xi_W}, \delta Z_{\xi_B}$)
Identical to SM.
If we introduce the wavefunction renormalization constants to the gauge symmetric states: SSB,

$$\xi_W = \xi_B = 1, \quad \delta Z_{\xi_W} = \delta Z_W, \quad \delta Z_{\xi_B} = \delta Z_B.$$

If we introduce them to the mass eigenstates,

$$\xi_W = \xi_Z = \xi_A = 1, \quad \delta Z_{\xi_W} = \delta Z_W, \quad \delta Z_{\xi_Z} = \delta Z_{ZZ}, \quad \delta Z_{\xi_A} = \delta Z_{AA}.$$

- Higgs sector ($\delta Z_{H_1}, \delta Z_{H_2}, \delta M_{H^\pm}^2, \delta \tan \beta$)
On-shell mass and residue conditions for H^\pm
On-shell mass condition for H^0
Decoupling of W_μ and H^\pm for on-shell H^\pm .

- chargino sector $(\delta\mu, \delta M_2, \delta\lambda^w, \delta Z_{\tilde{H}_1}, \delta Z_{\tilde{H}_2})$
On-shell mass condition for both $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^+$
Residue condition for $\tilde{\chi}_1^+$.
- neutralino $(\delta\lambda, \delta M_1)$
On-shell mass condition and the residue condition for $\tilde{\chi}_1^0$ (lightest neutralino)
- slepton sector $(\delta Z_L, \delta Z_R, \delta m_{\tilde{\tau}_L}^2, \delta m_{\tilde{\tau}_R}^2, m_{\tilde{m}_{\tilde{\tau}_{LR}}}^2)$
On-shell mass condition for $\tilde{\nu}, \tilde{\tau}_1, \tilde{\tau}_2$
Residue condition for $\tilde{\tau}_1$
Decoupling condition between $\tilde{\tau}_1$ and $\tilde{\tau}_2$ for on-shell $\tilde{\tau}_1$.
- squark sector $(\delta Z_L, \delta Z_R^t, \delta Z_R^b, \delta m_{\tilde{t}_L}^2, \delta m_{\tilde{t}_R}^2, \delta m_{\tilde{t}_{LR}}^2, (\delta m_{\tilde{b}_L}^2), \delta m_{\tilde{b}_R}^2, \delta m_{\tilde{b}_{LR}}^2)$
on-shell mass condition for \tilde{t}_i and \tilde{b}_i , $(i = 1, 2)$.
Residue condition on \tilde{t}_1 and \tilde{b}_1
Decoupling conditions on \tilde{q}_1 - \tilde{q}_2 ($q = t, b$) when \tilde{q}_1 is on-shell.
- charge
Thomson limit $(q_\mu \rightarrow 0, q^2 \rightarrow 0)$; $\frac{e^2}{4\pi} = \frac{1}{137.036}$
Ward identity \rightarrow no new constraint

5.3 Input

- mass of the conventional fermions
- $e, M_W, M_Z, M_{H^0}, \tan \beta$ (which fixes $M_{H^\pm}^2$)
- μ, M_2 (which determine the two masses of charginos.)
- M_1 (which determines all the masses of neutralinos.)
- All sfermion masses (which fix A_f)
- The following mass parameters are not independent from the input masses.

$$\begin{aligned}
 M_{h^0}^2 &= M_{H^0}^2 - \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}, \\
 M_A^2 &= M_{H^\pm}^2 - M_W^2, \\
 M_{G^\pm}^2 &= \xi_W M_W^2, \\
 M_{G^0}^2 &= (c_W^2 \xi_W + s_W^2 \xi_B) M_Z^2,
 \end{aligned}$$

Their pole masses are shifted from the above value by receiving radiative corrections.

- Three parameters μ, M_1 and M_2 fix the masses of two charginos and four neutralinos.

6 Other schemes

- Gauge sector

$$\begin{aligned} W_{\mu 0}^{\pm} &= Z_W^{1/2} W_{\mu}^{\pm}, \\ \begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix}_0 &= \begin{pmatrix} Z_{AA}^{1/2} & Z_{AZ}^{1/2} \\ Z_{AZ}^{1/2} & Z_{ZZ}^{1/2} \end{pmatrix} \begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix}. \end{aligned}$$

Three more wavefunction renormalization constants.

- Residue conditions for W^{\pm} and Z^0 ,
- Decoupling condition of A_{μ} - Z_{μ} for on-shell Z_{μ} .

- Higgs sector

Use A^0 in place of H^{\pm} to apply the on-shell mass condition.

- sfermion sector

First rotate

$$\begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}_0 = \begin{pmatrix} c_f & -s_f \\ s_f & c_f \end{pmatrix}_0 \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}_0 \quad (\text{Guasch et al})$$

or

$$\begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}_0 = \begin{pmatrix} c_f & -s_f \\ s_f & c_f \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}_0 \quad (\text{Boudjema et al})$$

Then introduce the wavefunction renormalization constants.

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}_0 = \begin{pmatrix} Z_{11}^{1/2} & Z_{12}^{1/2} \\ Z_{21}^{1/2} & Z_{22}^{1/2} \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}.$$

Three more wavefunction renormalization constants (lepton)

- On-shell mass and residue conditions for $\tilde{\nu}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$.
- Decoupling of $\tilde{\tau}_2$ - $\tilde{\tau}_2$ transition when one of the external leg is on-shell.

and five more wavefunction renormalization constants (squark)

- On-shell mass and residue conditions for \tilde{t}_1 , \tilde{b}_1 , \tilde{b}_2 .
- Decoupling condition for \tilde{q}_1 - \tilde{q}_2 ($q = t, b$) when \tilde{q}_1 or \tilde{q}_2 is on-shell.

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$$\delta\theta_q = \frac{1}{4}(\delta Z_{12} - \delta Z_{21}), \quad q = t, b$$

7 Comments

- Difficult to find the best renormalization scheme. unless some SUSY particle is discovered.
- \overline{DR} is simple as renormalization, but difficult to adjust the parameters in order to obtain desired physical masses.
If SUSY particles will be discovered (which we expect), the on-shell renormalization is more suitable than \overline{DR}
- Process dependent renormalization conditions
- We need several interfaces, such as
physical mass \rightarrow lagrangian parameter
unification value \rightarrow low energy parameters (CMSSM)