EW radiative corrections to WWγ production at ILC/LHC

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Outline

- Introduction
- $W^+W^-\gamma$ production at ILC

  Input parameter scheme: $\alpha(0)\ or\ G_F$?

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  ISR(initial-sate radiation) effect
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  NLO EW corrections

  W boson pair decay with spin correlation
- Summary

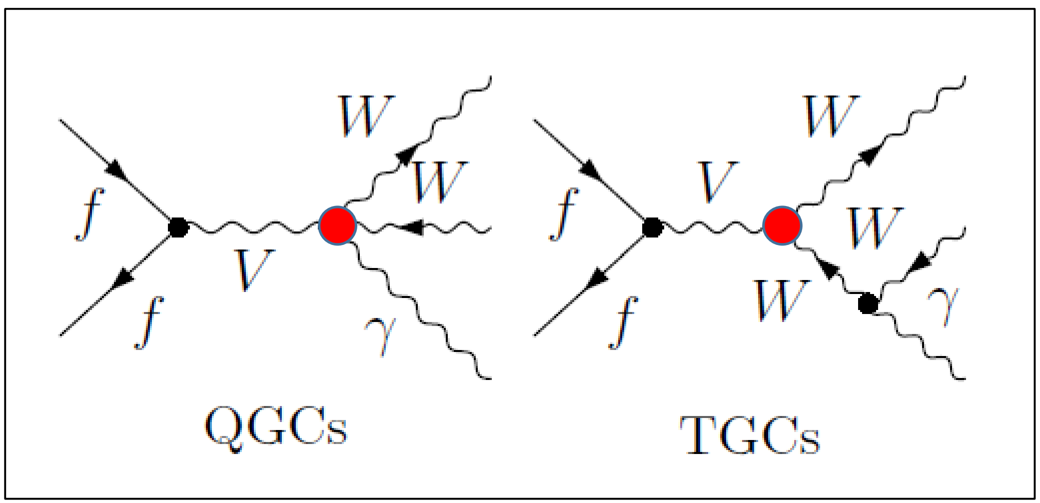
Introduction

physical motivation:

- Related to EW triple and quartic gauge couplings

  TGCs :$WW\gamma$, WWZ   QGCs:$WW\gamma\gamma$, $WWZ\gamma$

- Sudakov effects   significant EW effect at high energies



Introduction

Current status of VVV production(V= W, Z, γ):

- VVV production at LHC

  NLO QCD corrections complete

  Zγγ production with leptonic decays and triple photon production at NLO QCD *arXiv:1107.3149*

  NLO EW corrections WWZ

  NLO corrections to WWZ production at the LHC *arXiv:1307.7403*

- VVV production at ILC

  NLO EW corrections $\mathrm{WWZ}, \mathrm{ZZZ}, \mathrm{Z}\gamma\gamma, WW\gamma$

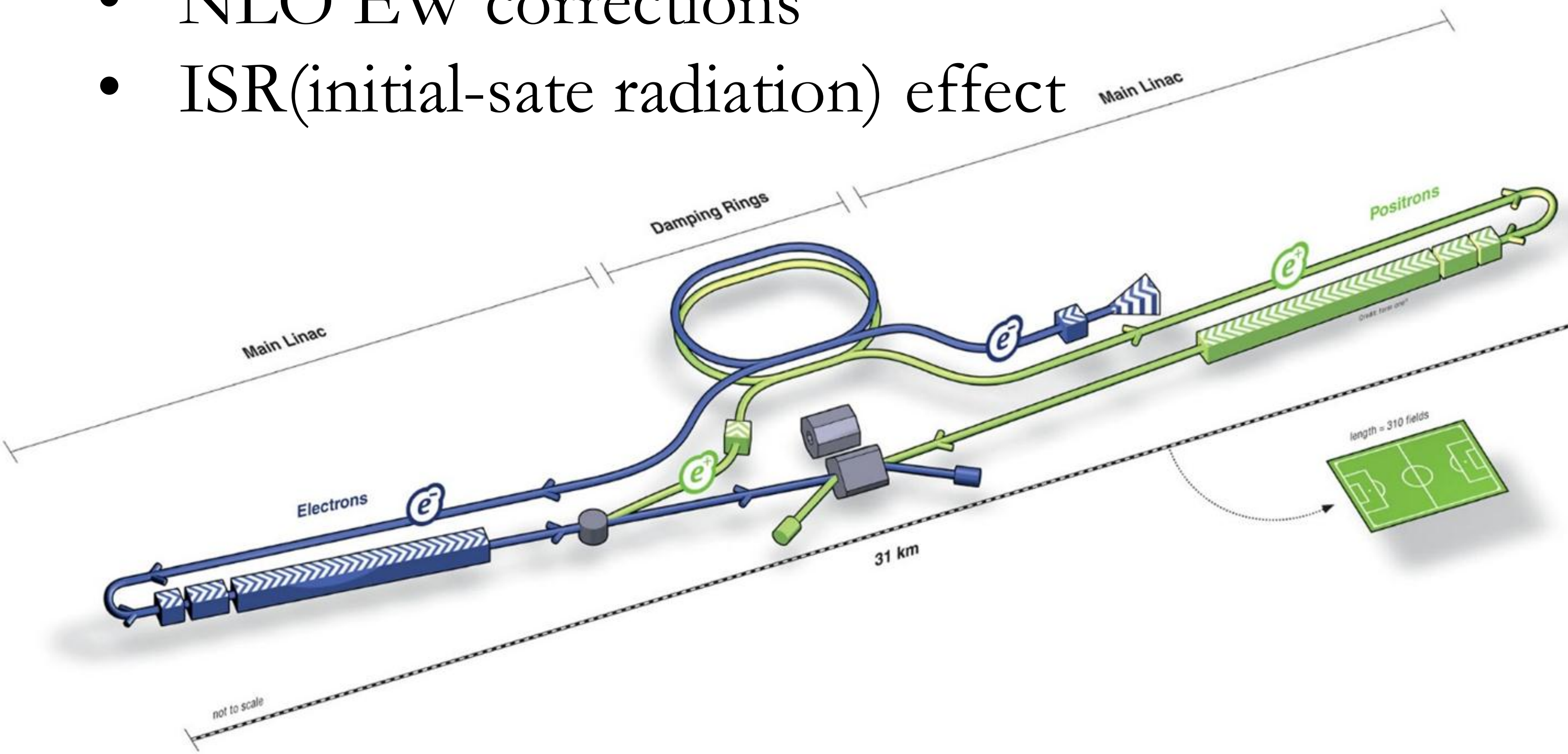
  NLO corrections to WWZ and ZZZ production *arXiv:0912.4234, 0807.0669, 0909.1064*

  Diphoton plus Z production at the ILC at $\mathcal{O}(\alpha^4)$ *arxiv: 1311.7240*

  Electroweak radiative corrections to $WW\gamma$ production at the ILC *arXiv:1409.4900*

EW corrections to $WW\gamma$ production at ILC

- Input parameter scheme: α_0 or G_μ?
- NLO EW corrections
- ISR(initial-sate radiation) effect



WWγ production at ILC

Input parameter scheme(α):

The choice of α:

- $\alpha(0) \sim 1/137$ — $\alpha(0)$-scheme
- $\alpha(M_Z) \sim 1/129$ — $\alpha(M_Z)$-scheme
- $\alpha_{G_F} \sim 1/132$ — G_F-scheme

Different choice of α differs of 2-6% at LO

For processes $\boldsymbol{\sigma_{LO} \propto O(\alpha^n)}$ and contain $\boldsymbol{l}$ **external** $\boldsymbol{\gamma}$

$\boldsymbol{n}$ EW couplings $n 2\delta Z_e^{\alpha(0)}$ → contain mass singularity term $\log(Q^2/m_f^2)$

$\boldsymbol{l}$ external $\boldsymbol{\gamma}$ $\boldsymbol{l}\delta Z_{AA}$ → exactly cancels the mass singularity term in $\boldsymbol{l}2\delta Z_e$

The appropriate input scheme: $\boldsymbol{\alpha(0)^l \alpha(M_Z)^{n-l}}$ or $\boldsymbol{\alpha(0)^l \alpha_{G_F}^{n-l}}$

WWγ production at ILC

Input parameter scheme(α):

In our calculation,the couplings related to the external photon are fixed at $\alpha(0)$ and the others α_{G_F}

$$\sigma_{LO}^{WW\gamma} \propto \alpha(0)\alpha_{G_F}^2 \qquad \Delta\sigma_{NLO}^{WW\gamma} \propto \alpha(0)^2\alpha_{G_F}^2$$

The coupling and corresponding renormalization constant in G_F-scheme:

$$\alpha_{G_F} = \frac{\alpha(0)}{1-\Delta r}$$

mass singularity terms canceled

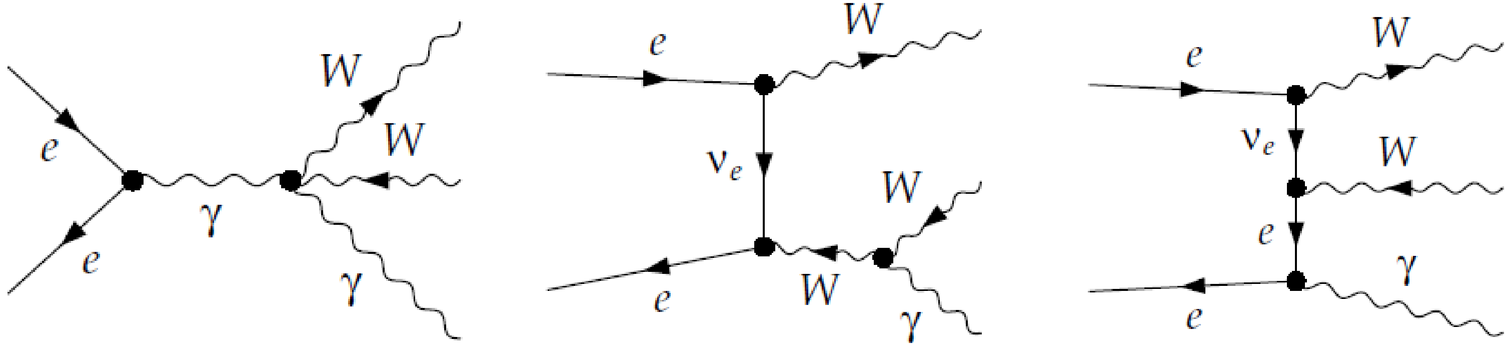
$$\delta Z_e^{\alpha_{G_F}} = \boldsymbol{\delta Z_e^{\alpha(0)} - \frac{1}{2}(\Delta r)_{1-loop}}$$

WWγ production at ILC

NLO EW corrections: $\sigma_{NLO} = \boldsymbol{\sigma_{tree}} + \Delta\sigma_{virtual} + \Delta\sigma_{real}$

- **Tree(18 Feynman diagrams)** $\boldsymbol{\sigma_{tree} \sim |\mathcal{M}_{tree}|^2}$

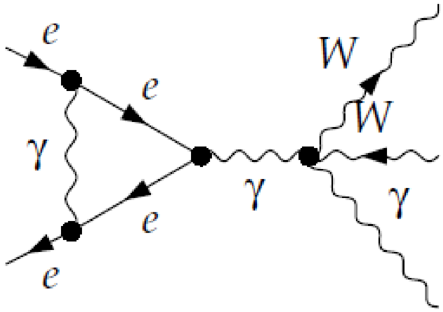
$P_T^{\gamma} > 15GeV, |y^{\gamma}| < 2.5$ to remove the infrared divergence at tree level

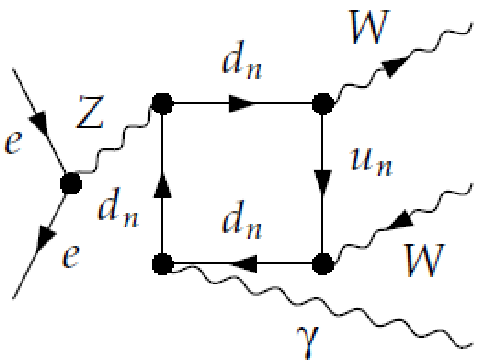


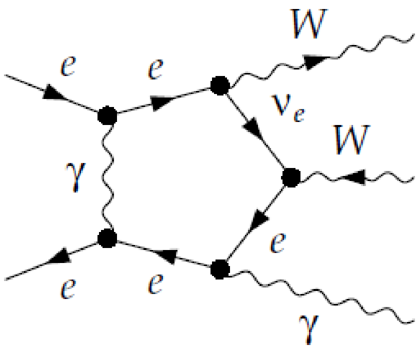
WWγ production at ILC

NLO EW corrections: $\sigma_{NLO} = \sigma_{tree} + \Delta\sigma_{virtual} + \Delta\sigma_{real}$

- **Virtual(2485 diagrams)** $\Delta\sigma_{virutal} \sim 2Re(\mathcal{M}^{*}_{tree}\mathcal{M}_{loop})$





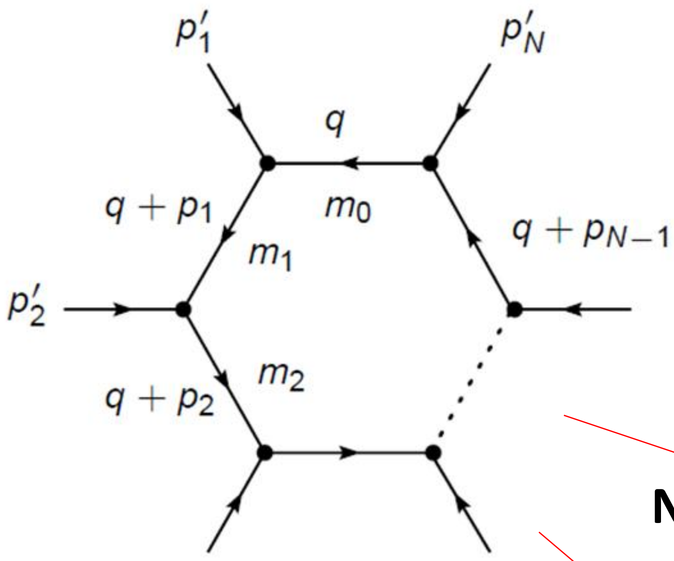


Up to five-point functions

WWγ production at ILC

NLO EW corrections: $\Delta\sigma_{virtual}$

- Evaluation of one-loop integrals (LoopTools-2.8)



N-point rank-M integral:

$$T^{N}_{\mu_1\ldots\mu_M} = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q\,\frac{q_{\mu_1}\ldots q_{\mu_M}}{D_1\ldots D_{N-1}}$$

Where $D_i = ((q+p_i\ )^2 - m_i^2)$

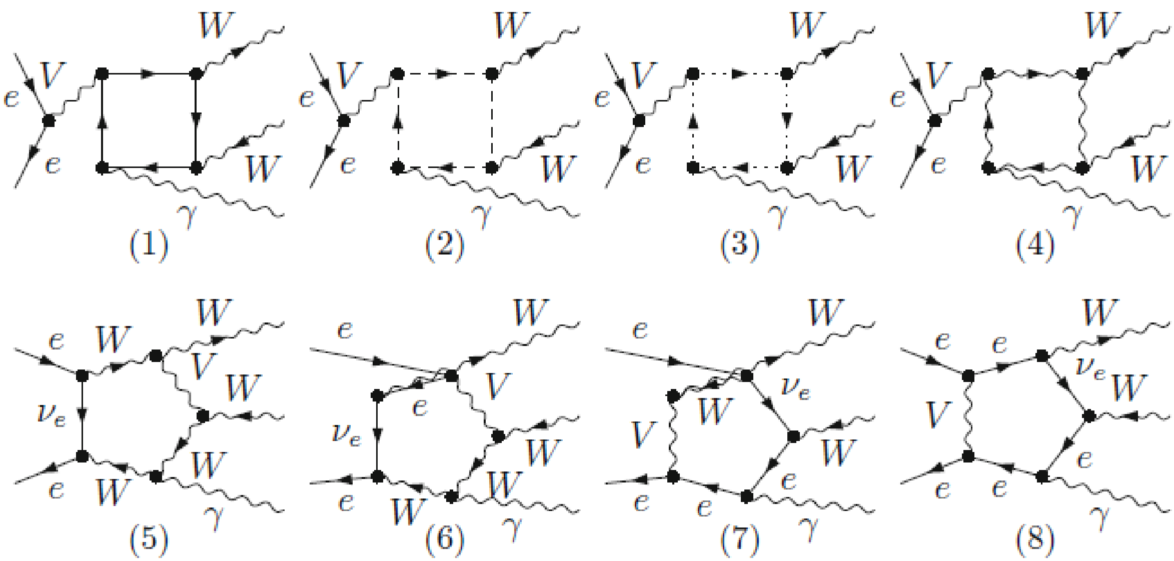
N$\leqslant 4$ → reduced to **scalar integrals** recursively (Passarino–Veltman method)

N =5 → decomposed into **4-point integrals** (the method of Denner and Dittmaier)

WWγ production at ILC

NLO EW corrections: $\Delta\sigma_{virtual}$

- **Numerical instability in loop calculation**



Problematic integrals:

4 or 5-point rank-4 tensor integrals

WWγ production at ILC

NLO EW corrections: $\Delta\sigma_{virtual}$

- **Numerical instability in loop calculation**

Reason I: small Gram determination

In Passarino–Veltman method, the rank-M tensor integral includes the tensor coefficients of the form:

$$T^{N}_{j_1 \ldots j_M} \sim \frac{N(p,m)}{(detG_N)^M} \qquad G_N = \begin{pmatrix} 2p_1p_1 & \cdots & 2p_1p_{N-1} \\ \vdots & \ddots & \vdots \\ 2p_{N-1}p_1 & \cdots & 2p_{N-1}p_{N-1} \end{pmatrix}$$

vanish $detG_N$, regular $T^{N}_{j_1 \ldots j_M}$

↓

large number cancelations in $N(p,m)$

↓

numerical instability

WWγ production at ILC

NLO EW corrections: $\Delta\sigma_{virtual}$

- **Numerical instability in loop calculation**

Reason I: small Gram determination

We developed the library **LoopTools-2.8**, which will use **quadruple precision** automatically when $\det(G_3)$ is small enough ,i.e.

$$\frac{\det(G_3)}{(2p_{max}^2)^3} < 10^{-5}$$

Then any $N \leq 5, M \leq 4$ loop integrals can be calculated numerically and stably.

WWγ production at ILC

NLO EW corrections: $\Delta\sigma_{virtual}$

- **Numerical instability in loop calculation**

Reason II: scalar one-loop 4-points integrals

- LoopTools-2.8: **FF package** (Version-a)
  the program implemented by Denner (Version-b)

Version-a/b alone, not ok, serious numerical problems

solution

repaired Version-b as our default version, when it fails, version-a is used

The **package Oneloop** is used to verify the correctness of our code

WWγ production at ILC

NLO EW corrections: $\Delta\sigma_{virtual}$

- **Works based on the modified LoopTools-2.8**

1.Diphoton plus Z production at the ILC *Eur.Phys.J. C 74,2739,*
*Zhang Yu, Guo Lei, Ma Wen-Gan, Zhang Ren-You, **Chen Chong**, Li Xiao-Zhou*

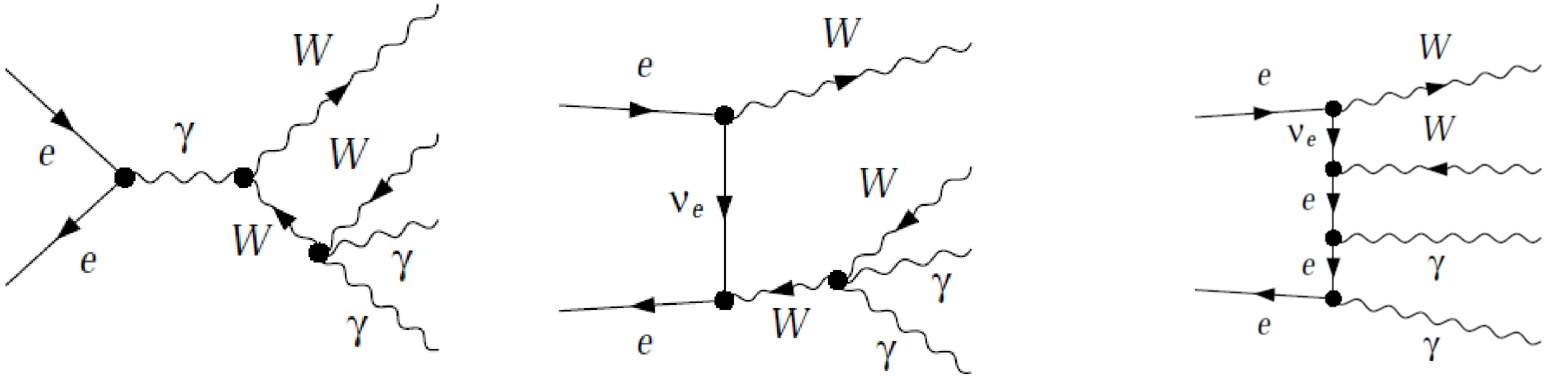
2. Possible effects of the large extra dimensions on ZZW production at the LHC
Mod. Phys. Lett. A, Vol. 29, No. 31 (2014) 1450153
***Chen Chong**, Guo Lei, Ma Wen-Gan, Zhang Ren-You, Li Xiao-Zhou, Zhang Yu*

3. QCD NLO and EW NLO corrections to ttH production with top decays at hadron collider
Phys. Lett. B, Volume 738, Pages 1–5
*Zhang Yu, Ma Wen-Gan, Zhang Ren-You, **Chen Chong**, Guo Lei*

WWγ production at ILC

NLO EW corrections: $\sigma_{NLO} = \sigma_{tree} + \Delta\sigma_{virtual} + \Delta\sigma_{real}$

- **Real-r emission(138 diagrams)** $\Delta\sigma_{real} \sim |\mathcal{M}_{real}|^2$



WWγ production at ILC

NLO EW corrections: $\Delta\boldsymbol{\sigma}_{\boldsymbol{real}}$

- **Photon candidates**

Cambridge/Aachen (C/A) jet algorithm:

- $\boldsymbol{R_{\gamma\gamma} < 0.4}$, we merge them into one new photon with momentum $p_{ij,\mu} = p_{i,\mu} + p_{j,\mu}$ **one-photon events** ,$P_T^{\gamma} > 15GeV, |y^{\gamma}| < 2.5$

- $\boldsymbol{R_{\gamma\gamma}} = \sqrt{(\Delta \boldsymbol{y}^2 + \Delta\emptyset^2)} \geq \boldsymbol{0.4}$, **two-photon events**, at least one photon satisfy the above cuts

WWγ production at ILC

NLO EW corrections: $\Delta\boldsymbol{\sigma}_{real}$

- **I: two cutoff phase space slicing (TCPSS) method**

Soft region $E_\gamma \leq \delta_s\sqrt{s}/2$ → **Soft IR divergence**, exactly canceled with $d\sigma_{virtual}$

hard region $E_\gamma > \delta_s\sqrt{s}/2$

- hard collinear region $cos\theta_{\gamma e} \geq 1-\delta_c$ → **quasi-collinear IR divergence** due to the smallness of electron mass
- hard non-collinear region $cos\theta_{\gamma e} < 1-\delta_c$ → **IR finite**

$$d\sigma_{real} = d\sigma_{soft}(\delta_s) + d\sigma_{h-coll}(\delta_s,\delta_c) + d\sigma_{h-noncoll}\,(\delta_s,\delta_c)$$

We take $\delta_c = \delta_s/50$ and check the cutoff independence in the range of $\delta_s \in [10^{-4}, 10^{-2}]$

WWγ production at ILC

NLO EW corrections: $\Delta\boldsymbol{\sigma_{real}}$

- **II: the dipole subtraction method**

$$\sigma_{NLO} = \sigma_{LO} + \int_m \left[d\sigma_{virt} + \int_1 \boldsymbol{d\sigma_{dipole}} \right] + \int_{m+1} [d\sigma_{real} - \boldsymbol{d\sigma_{dipole}}]$$

IR divergence canceled

approximates the divergent behavior of $d\sigma_{real}$ in all soft/collinear regions

The dipole terms are only needed in the singular region

parameter $\boldsymbol{\alpha}$:distinct regions neighboring a singularity and regions without need of a subtraction

We take $\boldsymbol{\alpha = 0.1}$ and check the $\boldsymbol{\alpha}$ independence in the range of $\alpha \in [10^{-3}, 1]$

WWγ production at ILC

NLO EW corrections: $\Delta\sigma_{real}$

TCPSS method

☺ clear physics picture

☹ Large number cancel between $d\sigma_{soft}, d\sigma_{h-coll}, d\sigma_{h-noncoll}$

Dipole subtraction method

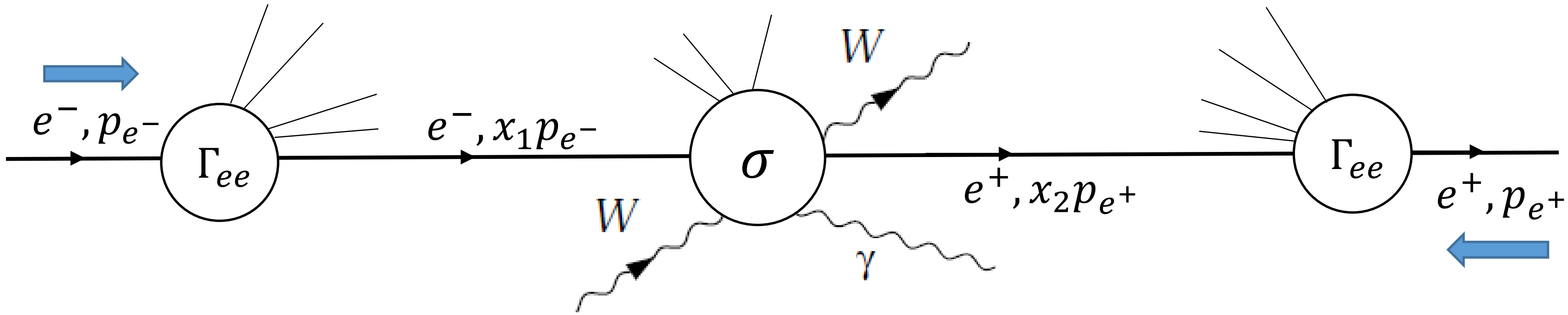
☹ There is a map of momentum between the dipole terms and real emission matrix

☺ Higher stability of numerical integration

In our calculation, we use TCPSS method taking its advantages of clear physics picture and the dipole subtraction method is used to verify the correctness of our numerical calculation.

WWγ production at ILC

ISR (initial-sate radiation) effect



ISR quasi-collinear IR divergences

- partially canceled by $\Delta\boldsymbol{\sigma}_{\boldsymbol{virtual}}$.
- The left leads to large terms $\alpha^n \log^n(m_e^2/Q^2)$ at the leading-logarithmic (LL) level

The structure function method

$$\int d\sigma_{ISR-LL} = \int_0^1 dx_1 \int_0^1 dx_2\, \Gamma_{ee}^{LL}(x_1, Q^2)\, \Gamma_{ee}^{LL}(x_2, Q^2) \int d\sigma(x_1 p_{e^-}, x_2 p_{e^+})$$

WWγ production at ILC

ISR (initial-sate radiation) effect

$$\int d\sigma_{ISR-LL} = \int_0^1 dx_1 \int_0^1 dx_2\, \Gamma_{ee}^{LL}(x_1, Q^2)\, \Gamma_{ee}^{LL}(x_2, Q^2) \int d\sigma(x_1 p_{e^-}, x_2 p_{e^+})$$

The explicit expression for $\Gamma_{ee}^{LL}(x, Q^2)$ up to $\mathcal{O}(\alpha^3)$ is given:

$$\Gamma_{ee}^{LL}(x, Q^2) = \frac{\exp\left(-\frac{1}{2}\beta_e\gamma_E + \frac{3}{8}\beta_e\right)}{\Gamma\left(1 + \frac{1}{2}\beta_e\right)} \frac{1}{2}\beta_e(1-x)^{\frac{1}{2}\beta_e - 1} - \frac{1}{2}\boldsymbol{\beta_e}(1+x)$$
$$-\frac{\boldsymbol{\beta_e^2}}{32}\left\{\frac{1+3x^2}{1-x}\ln(x) + 4(1+x)\ln(1-x) + 5 + x\right\}$$
$$-\frac{\boldsymbol{\beta_e^3}}{384}\left\{(1+x)[6Li_2(x) + 12\ln^2(1-x) - 3\pi^2]\right.$$
$$+\frac{1}{1-x}\left[\frac{3}{2}(1+8x+2x^2)\ln(x) + 6(x+5)(1-x)\ln(1-x)\right.$$
$$+12(1+x^2)\ln(x)\ln(1-x) - \frac{1}{2}(1+7x^2)\ln^2(x)$$
$$\left.\left.+\frac{1}{4}(39 - 24x - 15x^2)\right]\right\}$$

$$\beta_e = \frac{2\alpha}{\pi}(\ln\frac{Q^2}{m_e^2} - 1)$$

WWγ production at ILC

ISR (initial-sate radiation) effect

Double counting with NLO EW corrected results:

The **lowest-order** and **one-loop contributions** in $d\sigma_{ISR-LL}$ have to be subtracted to avoid double counting.

$$\int d\sigma_{ISR,LL,1} = \int_0^1 dx_1 dx_2 \left[\boldsymbol{\delta(1-x_1)\delta(1-x_2)} + \boldsymbol{\Gamma_{ee}^{LL,1}(x_1, Q^2)\delta(1-x_2)} + \boldsymbol{\delta(1-x_1)\Gamma_{ee}^{LL,1}(x_2, Q^2)}\right] \int d\sigma_0(x_1 p_{e^-}, x_2 p_{e^+})$$

The one-loop contribution to the structure function reads:

$$\Gamma_{ee}^{LL,1}(x, Q^2) = \frac{\beta_e}{4}\left(\frac{1+x^2}{1-x}\right)_+$$

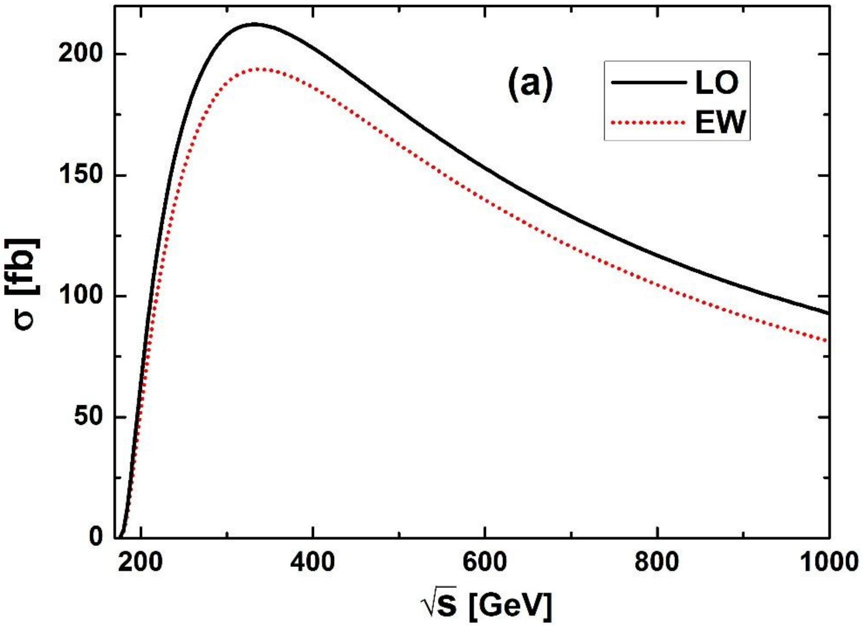
We call the subtracted ISR effect as the contribution of high order ISR effect (**h.o.ISR**)

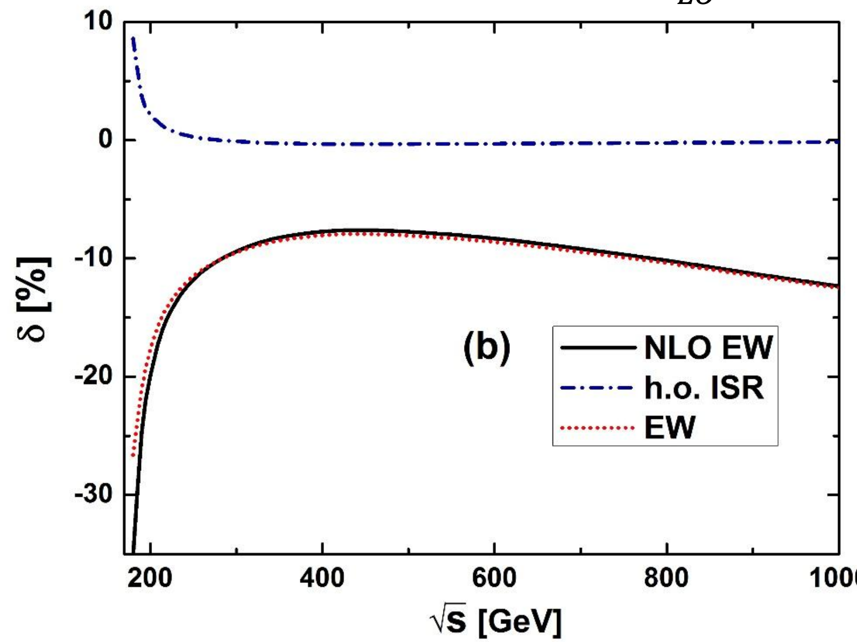
WWγ production at ILC

Total cross-section

$\sigma_{EW} = \sigma_{LO} + \Delta\sigma_{NLO} + \Delta\sigma_{h.o.ISR} \quad \mu_r = M_W$

The relative corrections are defined as $\delta = \frac{\sigma - \sigma_{LO}}{\sigma_{LO}}$



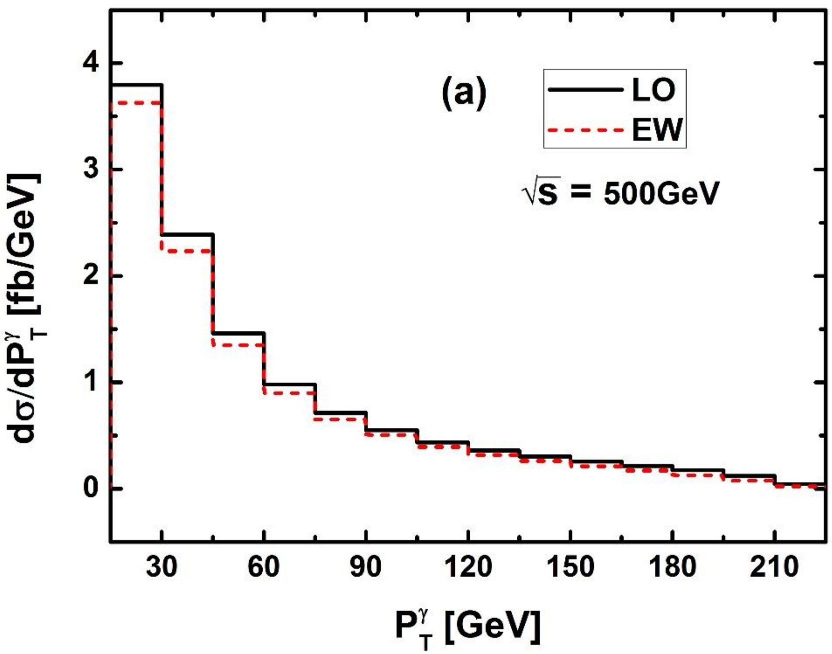


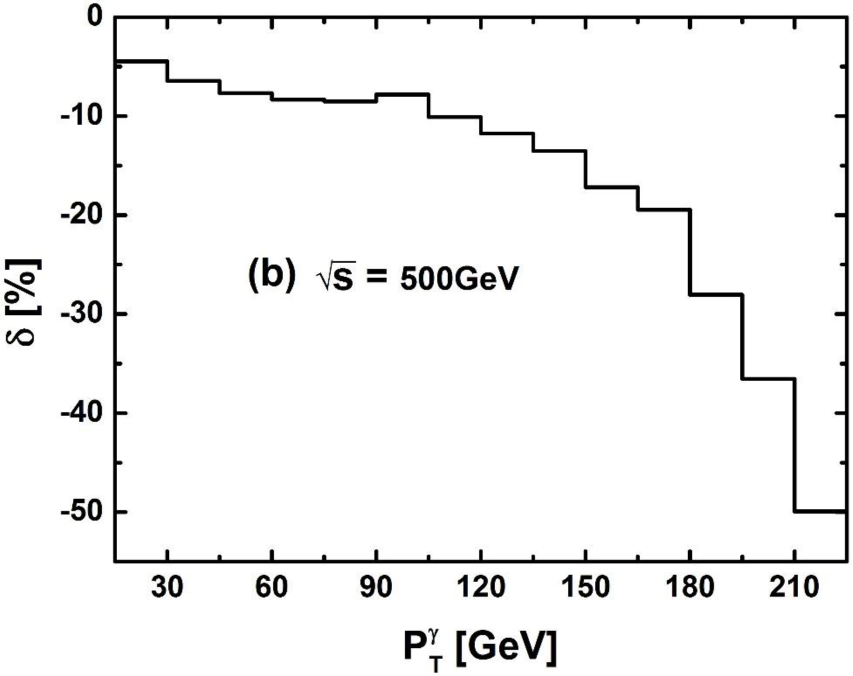
- In the vicinity of the threshold, $|\delta_{NLO}|$ is very large → Coulomb singularity effect
- At high energies, $|\delta_{NLO}|$ is also significant and goes up slowly with the increase of $\sqrt{s}$ → the Sudakov logarithms $\alpha\log^2(s/M_W^2)$
- The h.o.ISR effect is obvious only near the threshold

WWγ production at ILC

The P_T^{γ} distributions:

$$\sigma_{EW} = \sigma_{LO} + \Delta\sigma_{NLO} + \Delta\sigma_{h.o.ISR}$$

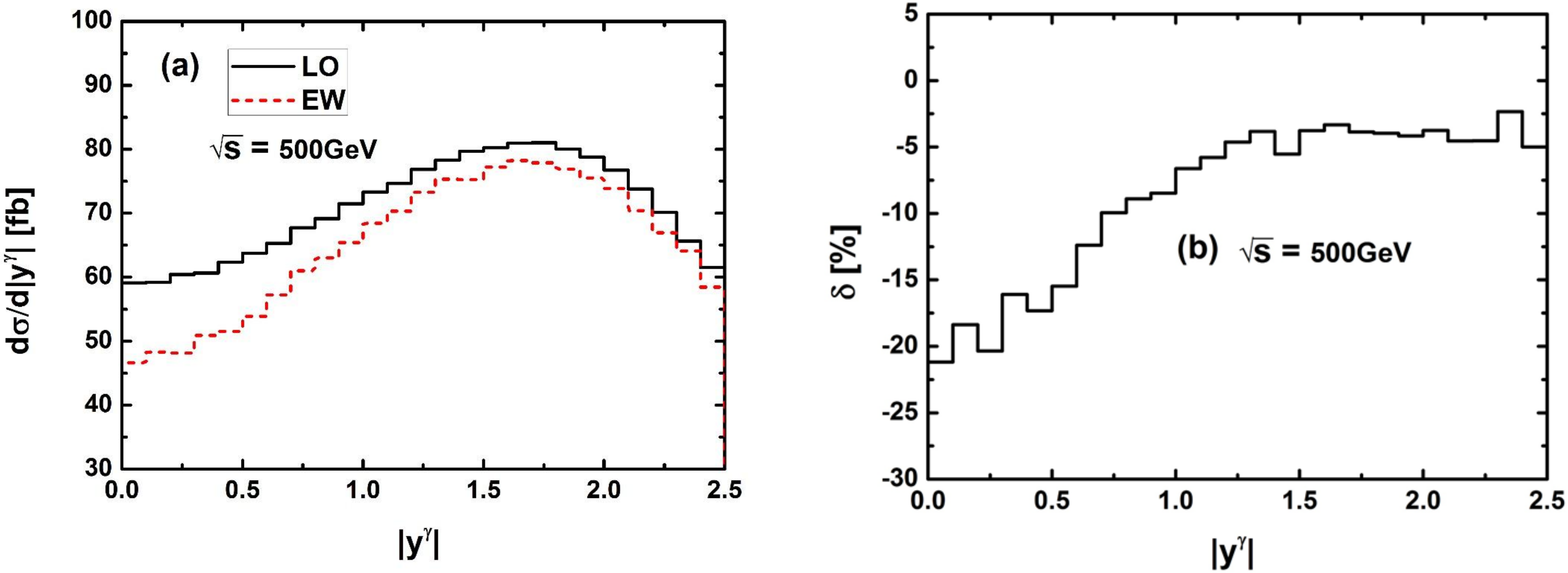




- EW correction **suppresses** the LO result
- At the end of P_T^{γ} distributions, $|\delta_{EW}(P_T^{\gamma})|$ becomes to be very large

WWγ production at ILC

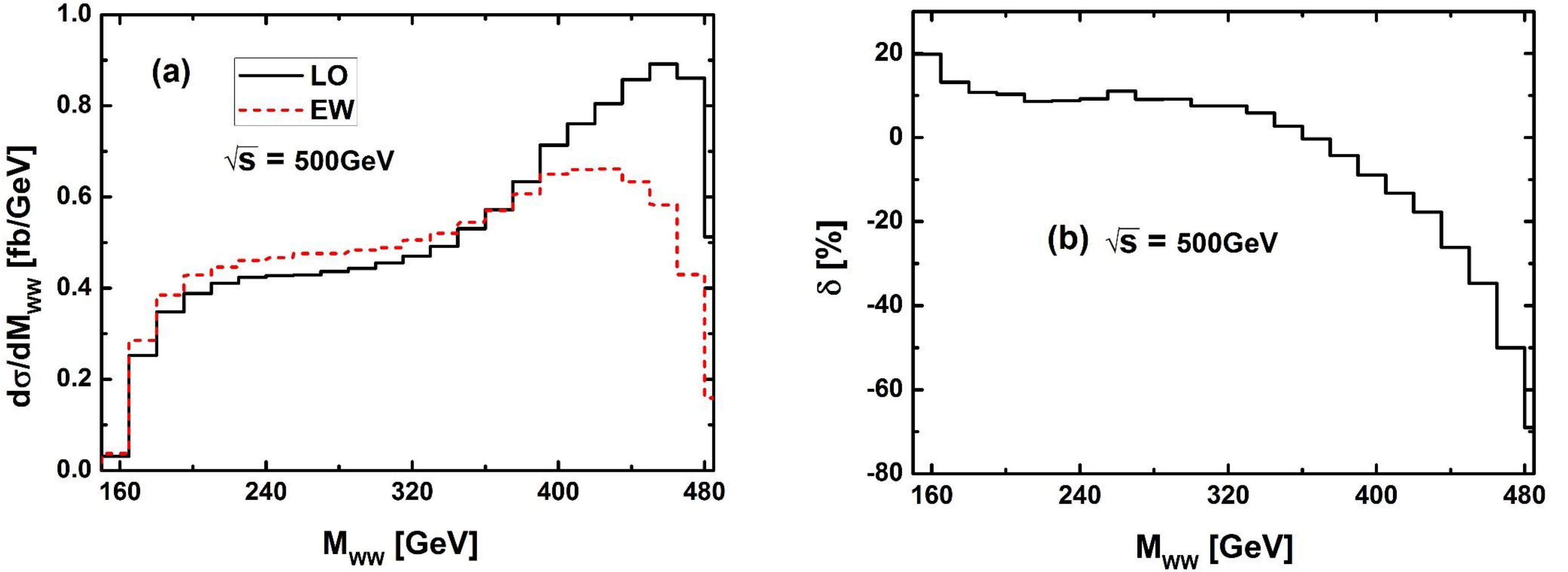
The $|y^{\gamma}|$distributions: $\sigma_{EW} = \sigma_{LO} + \Delta\sigma_{NLO} + \Delta\sigma_{h.o.ISR}$



- The outgoing γ are **bilateral symmetry** in the forward and backward hemisphere.
- EW correction **suppresses** the LO result
- $|\delta_{EW}(y^{\gamma})|$ reach its maximum in the central rapid region

WWγ production at ILC

The M_{WW} distributions: $\sigma_{EW} = \sigma_{LO} + \Delta\sigma_{NLO} + \Delta\sigma_{h.o.ISR}$



- In the region where M_{WW} is relative small, EW correction **enhance** the LO result
- At the end of M_{WW} distributions, EW correction **suppresses** the LO result and $|\delta_{EW}(P_T^{\gamma})|$ becomes to be very large

EW corrections to $WW\gamma$ production at LHC

- NLO EW corrections
- W boson pair decay with spin correlation

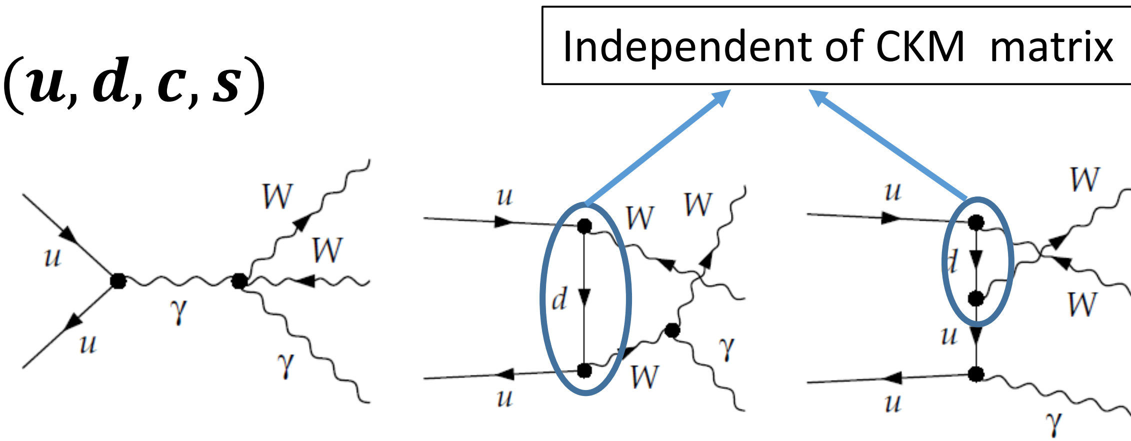
WWγ production at LHC

NLO EW corrections: subprocess

$q\bar{q} \to W^+W^-\gamma \quad q = (u, d, c, s)$

$\sigma_{LO} \propto \alpha(0)\alpha_{G_F}^2$

$\Delta\sigma_{EW} \propto \alpha(0)^2\alpha_{G_F}^2$

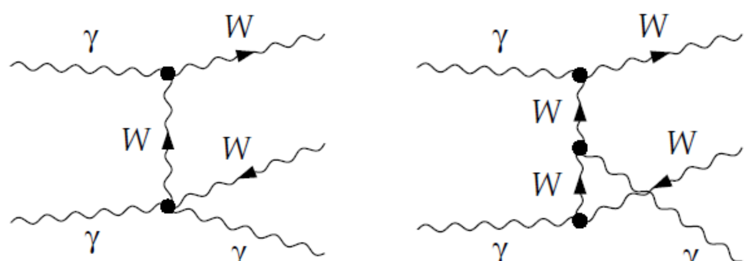


$b\bar{b} \to W^+W^-\gamma$

$\sigma_{b\bar{b}} \propto \alpha(0)\alpha_{G_F}^2$

$rr \to W^+W^-\gamma$

$\sigma_{rr} \propto \alpha(0)^3$



In our calculation:

- We neglect the masses of fermions except top quark
- The CKM matrix is set to be diagonal

WWγ production at LHC

NLO EW corrections: the difference from ILC

$e^+e^- \rightarrow W^+W^-\gamma$

- IR divergences : small photon mass m_γ
  the independence of m_γ
- Real-r emission : $e^+e^- \rightarrow W^+W^-\gamma\gamma$
  - soft IR divergence → $d\sigma_{virtual}$
  - **quasi**-collinear IR divergence → $d\sigma_{virtual}$; → h.o.ISR (the structure function method)

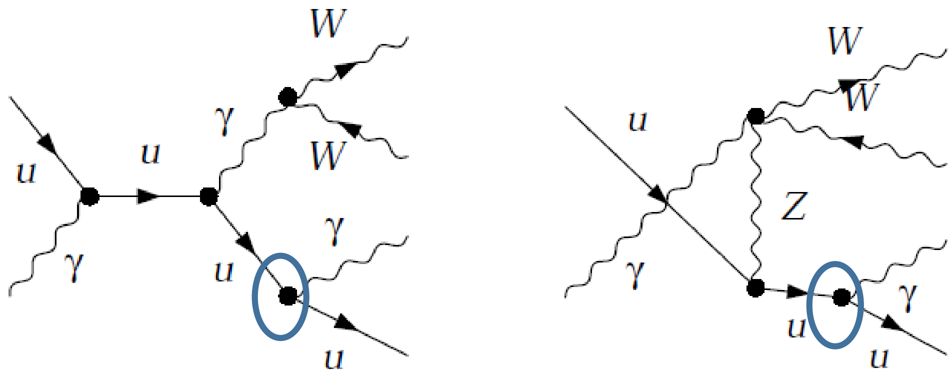
$q\bar{q} \rightarrow W^+W^-\gamma$

- IR divergences : dimensional scheme
  cancelation of the coefficients of $\frac{1}{\epsilon_{IR}}, \frac{1}{\epsilon_{IR}^2}$
- Real-r emission : $q\bar{q} \rightarrow W^+W^-\gamma\gamma$
  - soft IR divergence → $d\sigma_{virtual}$
  - initial-collinear IR divergence → $d\sigma_{virtual}$; → $d\sigma_{PDF}$
- **Real-q emission :** $q\gamma \rightarrow W^+W^-q\gamma$
  - initial-collinear IR divergence → $d\sigma_{PDF}$
  - final-collinear IR divergence → Smooth cut / **fragmentation**

WWγ production at LHC

NLO EW corrections

Real-q emission : $\boldsymbol{q\gamma \rightarrow W^{+}W^{-}q\gamma}$



- $\boldsymbol{R_{\gamma-jet} \geq 0.5}$ $P_T^{\gamma} > 15GeV, |y^{\gamma}| < 2$ → Non-final-collinear region
- $\boldsymbol{R_{\gamma-jet} < 0.5}$ $P_T^{\gamma+j} > 15GeV, \left|y^{\gamma+j}\right| < 2, z_{\gamma} = \frac{E_{\gamma}}{E_{\gamma}+E_{j}} > 0.9$ r-jet recombined

Phase space slicing method $\boldsymbol{S_{\gamma j} \leq \delta_c S_{12}}$ $S_{\gamma j} > \delta_c S_{12}$

The final collinear IR divergence is canceled by the bare fragmentation function:

$$D_{q\to\gamma}^{\mathrm{bare,DR}}(z_\gamma) = \frac{\alpha Q_q^2}{2\pi}\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{\mu_{\mathrm{F}}^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}P_{ff}(1-z_\gamma) + D_{q\to\gamma}(z_\gamma,\mu_{\mathrm{F}}),$$

WWγ production at LHC

Total cross-section

Table: tree level results

	σ[fb]	δ[%]
LO	189.06(2)	
$b\bar{b}$	3.2324(3)	1.71
rr	8.099(3)	4.28

- **Scheme-I(inclusive)** $P_T^\gamma > 15GeV, |y^\gamma| < 2$

For real-r emission , at least one photon could pass the above cuts.

- **Scheme-II(exclusive)** $P_T^\gamma > 15GeV, |y^\gamma| < 2, P_T^j < 50GeV$

For the real-r emission, only one photon could pass the above cuts.

Table: NLO EW corrections

		Scheme-I		Scheme-II	
		σ[fb]	δ[%]	σ[fb]	δ[%]
EW	$q\bar{q}$	-8.954(7)	-4.74	-9.849(7)	-5.21
	$q(\bar{q})\gamma$	13.385(4)	7.08	1.284(5)	0.68
total		4.430(9)	2.34	-8.564(9)	-4.53

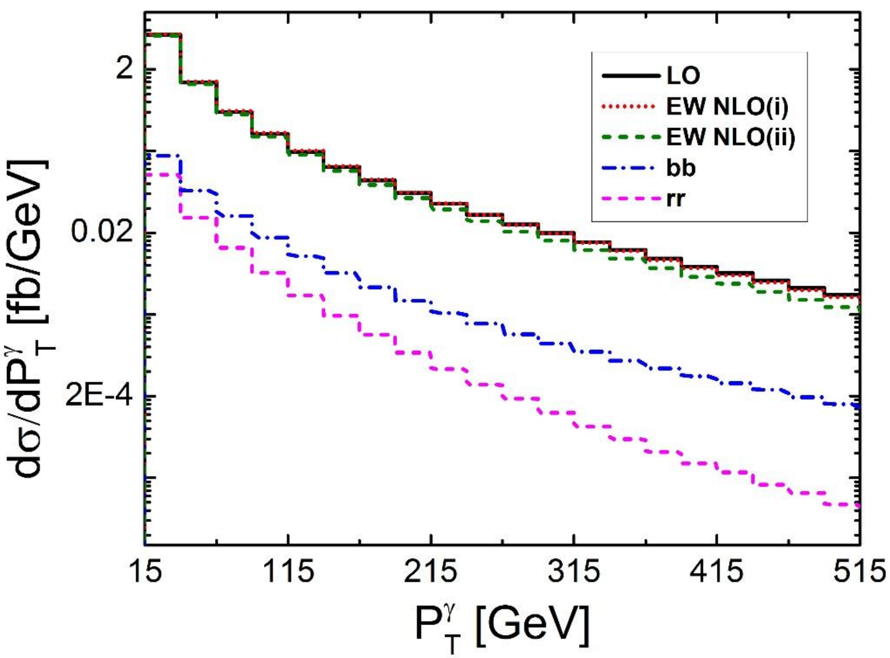
$$\delta = \frac{\sigma - \sigma_{LO}}{\sigma_{LO}}$$

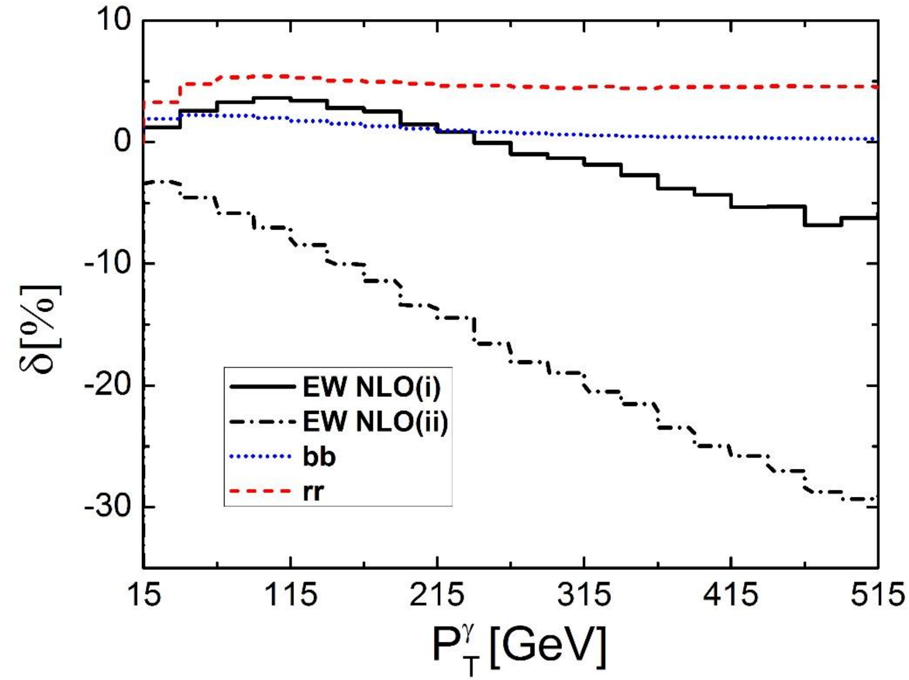
NNPDF23_lo_as_0119_qed

$\mu_f = \mu_r = M_W$

WWγ production at LHC

The P_T^{γ} distributions:

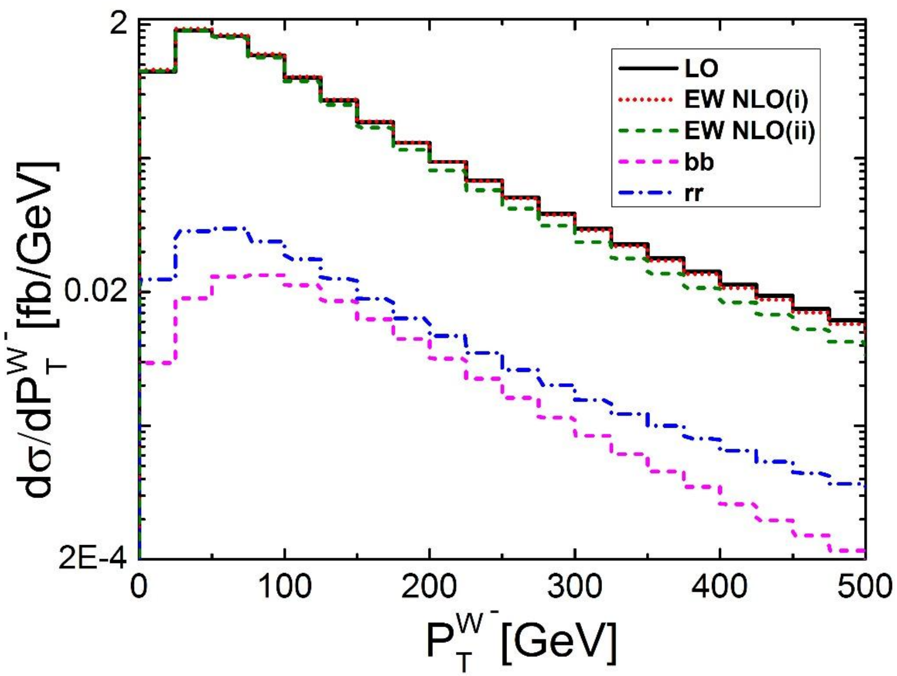


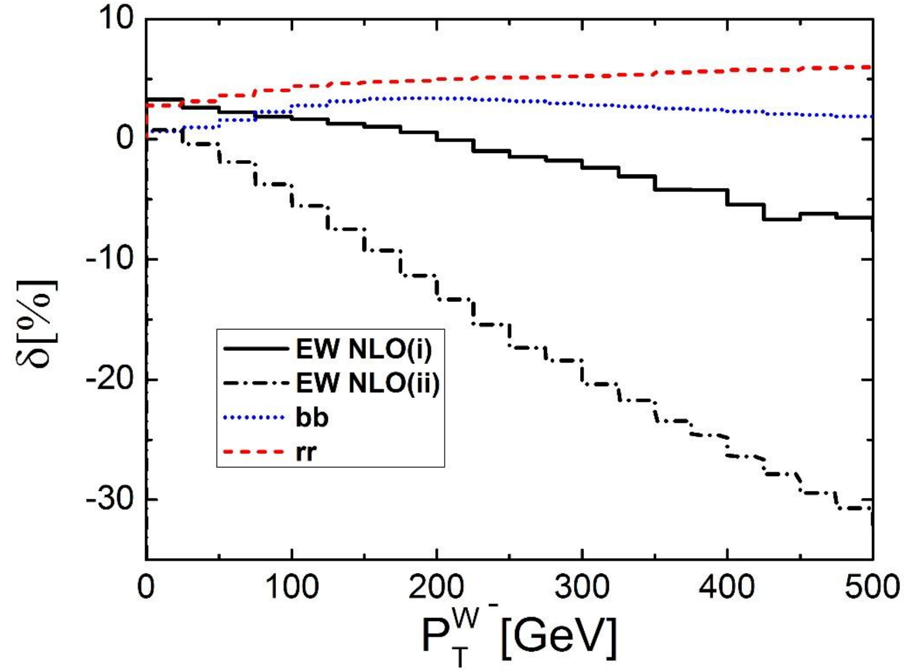


- In scheme-I, EW correction is small
- In scheme-II, $|\delta_{EW}(P_T^{\gamma})|$ becomes to be very large at the end of P_T^{γ} distributions

WWγ production at LHC

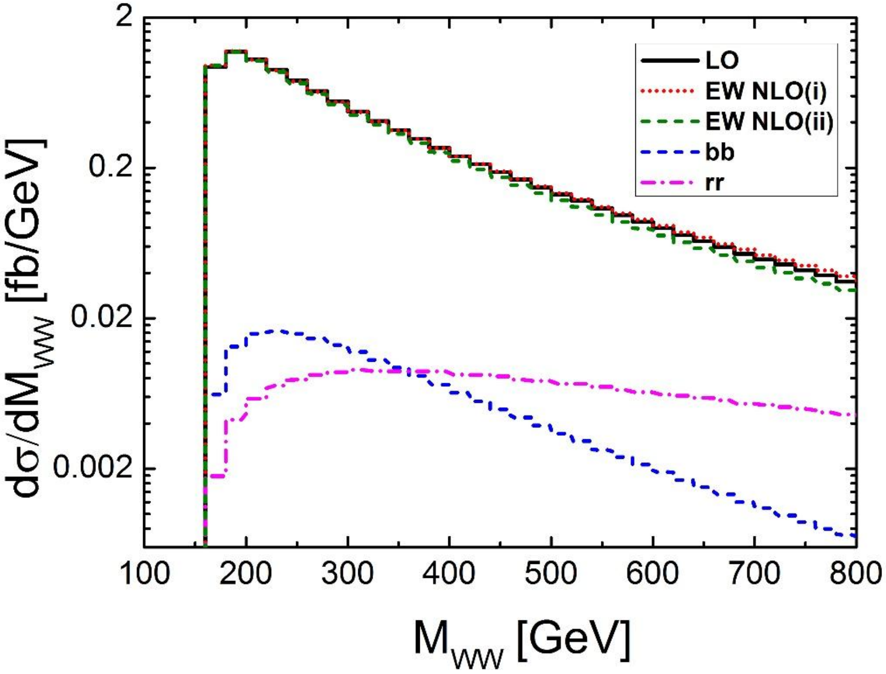
The $P_T^{W^-}$ distributions:

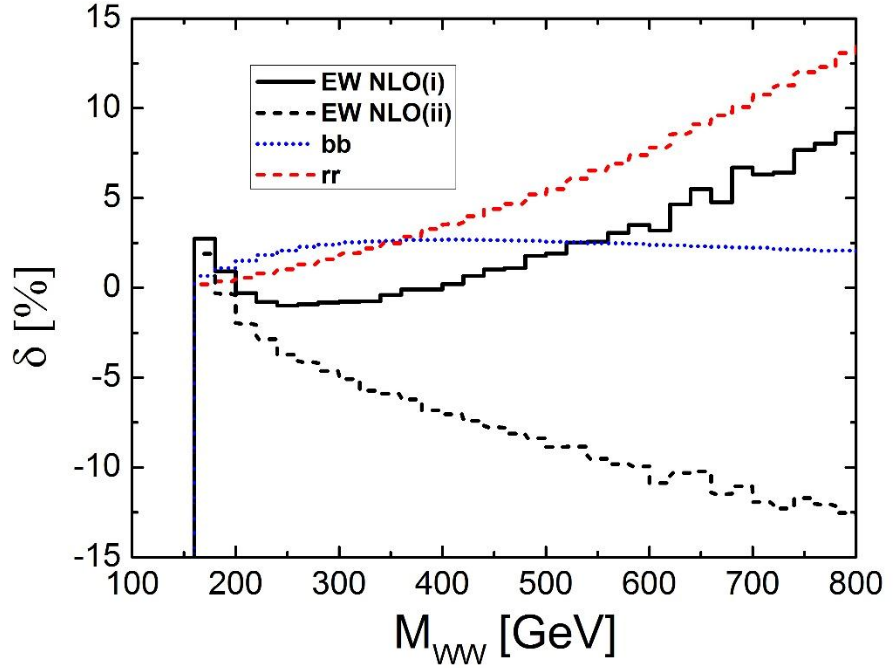




WWγ production at LHC

The M_{WW} distributions:





WWγ production at LHC

W boson pair decay with spin correlation :

$$\sigma_{ab\to VX\to f\bar{f}'X} = \frac{1}{2s_{ab}}\int d\Phi_{f\bar{f}'X}\left|\mathcal{M}_{ab\to VX\to f\bar{f}'X}\right|^2$$

$$\underset{\Gamma_V\to 0}{\sim} \frac{1}{2s_{ab}}\int d\Phi_{VX}\sum_{\lambda,\lambda'=0,\pm1}\mathcal{M}_{ab\to VX}(\lambda')^*\,\mathcal{M}_{ab\to VX}(\lambda)$$

$$\times\frac{1}{2M_V\Gamma_V}\int d\Phi_{f\bar{f}'}\left(\hat{k}_1,\hat{k}_2\right)\mathcal{M}_{V\to f\bar{f}'}(\lambda')^*\mathcal{M}_{V\to f\bar{f}'}(\lambda)$$

the phase space of the decay fermions is integrated over **completely**

$$2M_V\Gamma_{V\to f\overline{f'}}\delta_{\lambda\lambda'}$$

Narrow width approximation(NAW):

$$\sigma_{ab\to VX\to f\bar{f}'X} \sim \frac{1}{2s_{ab}}\int d\Phi_{VX}\sum_{\lambda=0,\pm1}\left|\mathcal{M}_{ab\to VX}(\lambda)\right|^2 BR_{V\to f\overline{f'}}$$

To explore the distributions in the fermion kinematics, we use the expressions before integral. (**NAW with spin correlation**)

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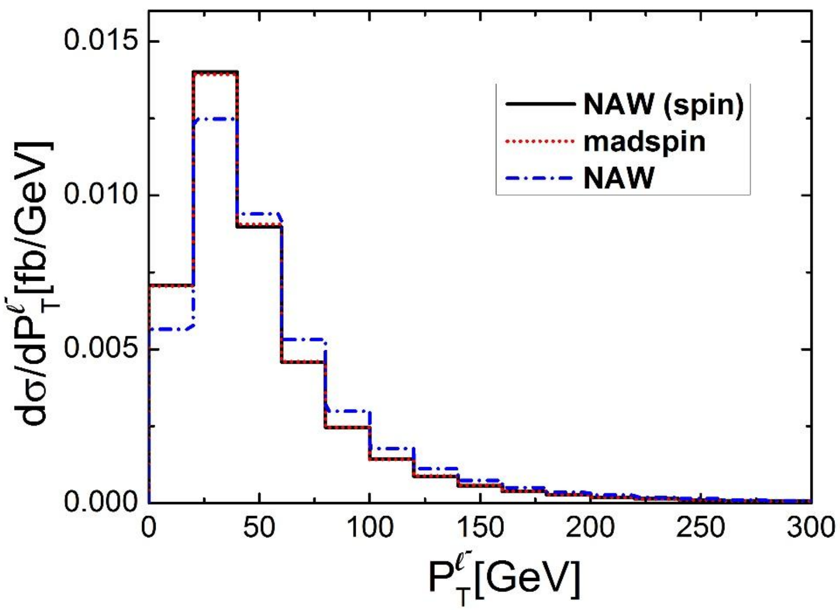
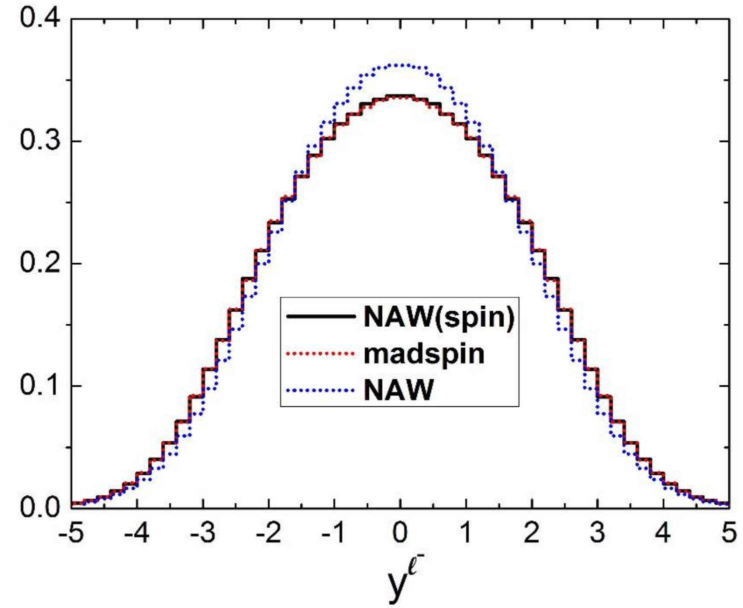
An example: $u\,\bar{u} \rightarrow W^{-}W^{+}r \rightarrow l^{-}\overline{\nu_l}l^{+}\nu_l r$ $\quad P_T^r > 15GeV, |y^r| < 2$

To verify the correctness of our code, we check our results with MadSpin.





Summary

- We present the complete NLO EW corrections to $e^+e^- \to W^+W^-\gamma$ as well as the h.o.ISR effect.

$$\sigma_{EW} = \sigma_{LO} + \Delta\sigma_{virt} + \Delta\sigma_{real} + \Delta\sigma_{h.o.ISR}$$

Modified LoopTools-2.8

TCPSS method/ dipole subtraction method

Structure function method

- We calculate the NLO EW corrections to pp $\to W^+W^-\gamma$.

$$\sigma_{EW} = \sigma_{LO} + \Delta\sigma_{virt} + \Delta\sigma_{real-r} + \Delta\sigma_{real-q} + \sigma_{b\bar{b}} + \sigma_{\gamma\gamma}$$

PSS method, fragmentation function

We also perform the leptonic decay of W boson pair with spin correlation, the code has been finished and the program is running for the final results.

Thank you for attention.